Math 10C

**Chapter 1 – Measurement**

**Review and introduction to basic shapes**

B E D M A S – this is the order of operations that MUST be used to solve an equation.

done at same time

Eg. Evaluate the following:

1. 

2. 

3. 

However, when we are solving for a variable in an equation, we reverse the operations of BEDMAS to get our variable isolated on one side of the equation. REMEMBER!! Whatever you do to one side of the equation, you MUST do to the other side of the equation.

Eg.  Solve for r.

Because the r is multipled by 3, I must divide both sides by 3 to solve for r.



5 = r

Even if there is more going on in the equation, always reverse the BEDMAS order when isolating a variable.

Eg.  Solve for h.

Well, the variable I want to solve for is in the second term so, the first thing I have to do is get the term with the variable I want all by itself on one side of the equation (BEDMAS – the last 2 letters are AS so I need to get rid of any addition or subtraction first)

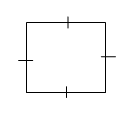


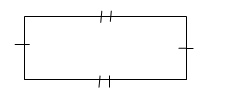
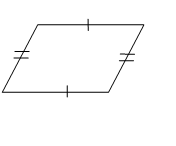
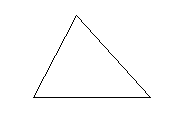
Now I can divide out the coefficients that are being multiplied to the term that I want. I do the opposite which is division. (BEDM – division and multiplication are the next 2 letters!)

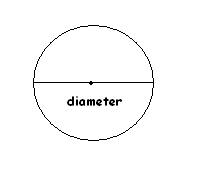


Now that I have isolated the variable, I am done!

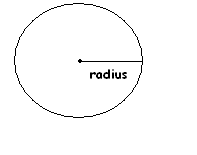
**Basic Shapes (REVIEW – handout for students)**

****

* **Polygon –** a closed shape that consists of line segments.
* **Quadrilateral -** a shape with four sides, such as a **square, rectangle, parallelogram, or trapezoid.**
* **Square**  - all sides are of an equal length
* **Rectangle** – two sides are the same length, and the other two are the same length. We usually call this **width** and **length**
* **Parallelogram** – usually described as a “slanted” rectangle. **Parallel** means that two (or more) lines will never cross. Squares and rectangles are technically parallelograms.
* **Trapezoid -** a four sided shape with only one set of parallel lines.
* **Triangle –** Three sided figure.
* **Circle -** All the points on a circle are equidistant (the same distance) from the CENTER of the circle.



* A line that passes through the center of a circle and touches the edge of the circle on both sides is called the diameter.
* A line that starts at the center of the circle and touches an outside edge is called the radius.

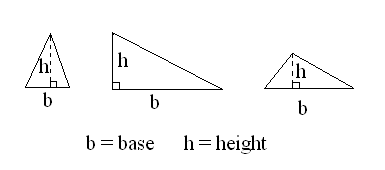
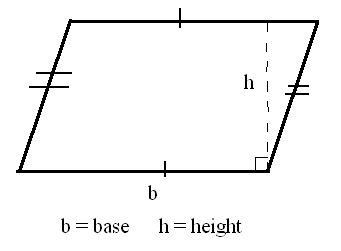


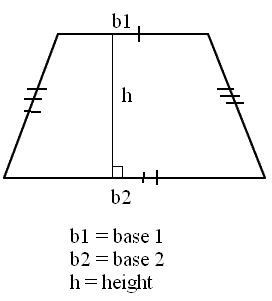
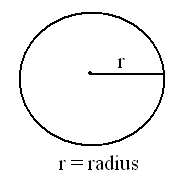
* The **circumference** of a circle is the perimeter of the circle. It can be calculated with the formula : 

Where C = circumference (perimeter)

 = “pi” a constant that is 3.14….

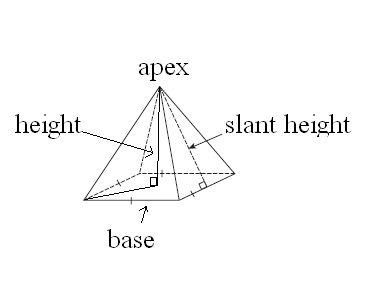
d = diameter

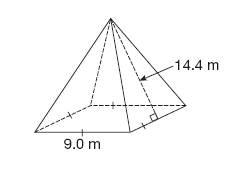
* **Perimeter** – the distance around the outside of an object. You can calculate the perimeter of a shape by adding up the lengths of each side.
* **Area -** is the surface of a 2-D object. A good way to visualize area is to imagine entirely coloring in between the lines of a shape; when you have done this, you have coloured the area.
* When we calculate the area, we are finding the “square” units of the shape.
* **Area Square** = side x side = s x s OR **s2**
* **Area Rectangle** = length x width = **l x w**
* The **height** of an object is the perpendicular distance from the base of a polygon to an opposite vertex.
* **Perpendicular** – two lines that form a right (900) angle. For example, a corner is a 900 angle.
* **Vertex** – the point where two or more lines meet.
* **Area triangle** = (base x height) OR **b x h**
* **Area Parallelogram =** base x height OR **b x h**

* **Area Trapezoid =** (base + opposite side) x height OR **(b1+b2) x h**
* **Area Circle =** x radius x radius OR r 2
* A **right pyramid** is an object that has triangular faces and a base that is a polygon. The shape of the base determines that name of the pyramid.

Each side of a prism/pyramid is called a **face –** a 2-D object that forms a flat surface of a 3-D object.

* The **surface area** of a 3-D object is the **sum** of all the **areas** of the faces of the object.
* Surface area can be easy to calculate if you are able to visualize the 2-D shapes (faces) that make up the 3-D object.



**Ex 1.**

* How many faces are there? **5**
* What shapes do you see? **4 triangles, 1 square**
* Ex1. Here are the dimensions of the pyramid. Calculate the surface area.
  + Area of triangular face **= ATriangle = ½ bh = ½ (9.0m)(14.4m) = 64.8 m2**
  + Area of square base **= ASquare = s2 = 9.0m2 = 81 m2**
  + **The SA of the pyramid is:**
  + SA = 4(64.8m2) + (81 m2) **= 340.2m2**

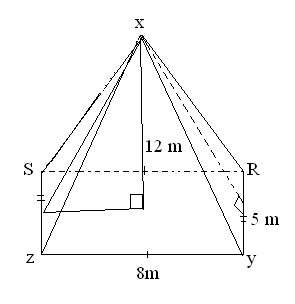
**\**Explain why we multiplied 64.8m2 by 4*\***

**\*NOTE:** The **lateral area** is the area of the triangular faces of a pyramid.

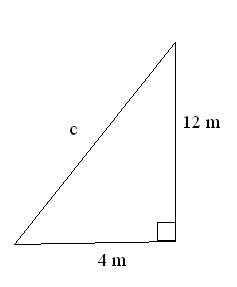
* Sometimes, what we learned in our trigonometry unit can be useful to help calculate the slant height of a right pyramid. Remember Pythagoras? **a2 + b2 = c2**

**Surface Area and Volume**

Ex. A right rectangular pyramid has base dimensions 5 m by 8 m and a height of 12 m. Calculate the surface area of the pyramid to the nearest square meter.



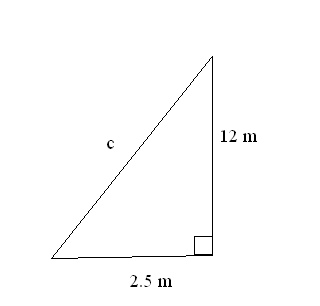
\*Sketch it out!\*

* Since this is a **rectangular pyramid**, there are two sets of congruent triangles; one set has a base of 8 m (XYZ and XSR), one set has a base of 5 m (XRY and XSZ)
* We can find slant height of each face using Pythagoras.
* We know that the height of the pyramid is from the **center** of the base; therefore, in the diagram above, to find the slant height of the triangle face with a base of 8 m, we will use:
  + *Explain to students how we calculated 4m*

c = m

The area of XRY and XSZ can now be calculated:

A= ½ bh = ½ (8 ) () = 4m2



* Now, calculate the slant height of the triangle with a base of 5 m:
  + *Explain to students how we calculated 2.5 m*

c = 

The area of XYZ and XSR can now be calculated:

A = ½ bh = ½ (5) () = 2.5m2

Now, calculate the area of the base of the pyramid:

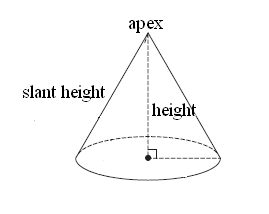
Abase = l x w = 8 x 5 = **40 m2**

Now, add together to get the total Surface Area of the pyramid.

SA = 2 **(4)()** + 2 (2.5)() + 40 = 202.481…m2 = **202 m2**

* We can use the formula **SA = ½ s (perimeter of base) + (base area)**
  + Where s = slant height

to calculate the surface are of any right pyramid with a regular polygon base. Read page 30 in your text book which describes how this formula is derived.

A right circular cone is a 3-D object that has a circular base and a curved surface. The ***height*** of the cone is the perpendicular distance from the apex to the base.

* The formula for the area of a right cone is



*🡪Which part is representing the area of the circle?* 

*🡪Which part is representing the lateral area?* 

Ex3. Calculate the area of a right cone with a radius of 37 inches and a slant height of 54 inches to the nearest inch:

 = (37)(54) + (37)2 = 10 577.74…in2 = **10 578 in2**

Ex 4. The later area of a cone is 416 cm2. The diameter of the cone is 18 cm. Determine the height of the cone to the nearest tenth of a centimeter.

Find the radius! = **9 cm**

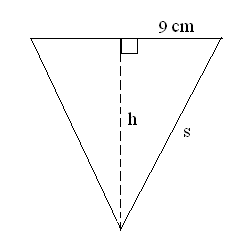
Alateral = 

416 = (9)s

**Divide both sides by 9**



s = 

s = 14.712…..

To determine the height of the cone, use the Pythagorean Theorem:

92 + h2 = s2

*Substitute for s*

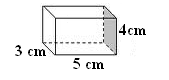
81 + h2 = (14.712….)2

*Solve for h*

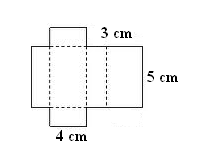
h = 

h = 11.6392…. = **11.6 cm**

A **prism** is a 3-D shape whose “bases” (or ends) are of the same size and shape and are parallel to one another. The base shape of a prism is usually described in the name. For example, a triangular prism has a base shape of a triangle.

A **cylinder** is a 3-D shape that has circles for the base. The most common cylinder is the soup can!

Ex1. Determine the surface area of the following rectangular prism:

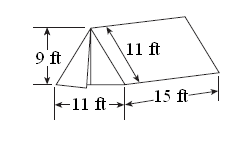
****

* The net would look like this:
* Notice how there are two sides that are 4 cm x 3cm, two sides that are 4 cm x 5 cm and two sides that are 3 cm x 5 cm? We can write a formula to describe the SA of a rectangular prism:
  + *SA = 2( l x w) + 2( l x h) + 2( w x h)*
    - Where *l* = length

*w =* width

*h* = height

* SA = 2 (5 x 3) + 2(5 x 4) + 2(3 x 4) = **94 cm2**



**Ex2. A)** Determine the surface area of the following tent:

* + What is the name of this prism? *Triangular prism*
  + How many faces? *Five*

Visualize the shapes that make up this prism:

Two triangles, b = 11 ft, h = 9ft

Three rectangles, l = 15 ft, w = 11 ft

Calculate the SA.

SA = 2( ½ bh) + 3(lw) = 2( ½ 9 x 11) + 3(15 x 11) = **594 ft2**

B) If canvas costs $0.40 per square foot, how much will the fabric cost to make the tent?

$0.40/ft2 x 594 ft2 = **$237.60**

C) What is the surface area of the exposed part of the tent?

*Explain to students how the bottom of the tent is “hidden” on the ground.*

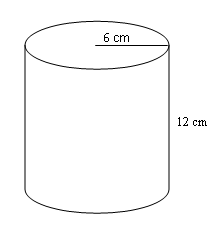
SA = 2( ½ 11 x 9) + 2( 15 x 11) = **429 ft2**

OR

Hidden part Area = 15 x 11 = 165 ft2

Total SA = 594 ft2

594 ft2 – 165 ft2 = **429 ft2**

Ex. A cylinder has a height of 12 cm and a radius of 6 cm. Determine the surface area to the nearest 10th of a cm2

**Sketch it!**



SA = 2(6x12) + 2(62) = **678.6cm2**

**Assignment:** M10C SA other shapes.docx

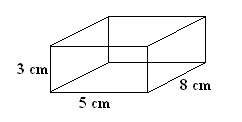
**1.4 Page 34 – 35** #5, 7, 8, 9, 10, 11, 13, 15, 16, 20, 21

**Volume of Right Pyramids and Right Cones**

**Volume** is the amount of space occupied by a 3-D object.

* We are most familiar with expressing volume as liters (L), milliliters (mL), gallons (gal), pints (pt), even cups when baking!
* *Did you know that 1 cup is approximately 240 mL? Then 2 cups must be about 500 mL -- half a litre!*
* It can be difficult to calculate the volume of 3-D objects in L or mL. These are called **capacity units.**  For this course, we will calculate volume in Imperial Units3 or Metric Units3.
* The **units** for volume are said to be “cubed” or to the power of 3. For example, cm3 or m3.
* To calculate the volume of a rectangular prism we use the formula….

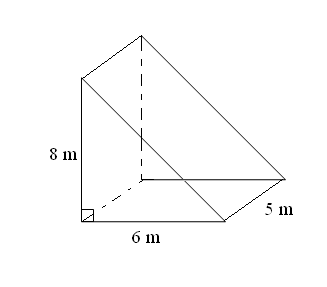
V = *lwh* Where l = length w = width h = height

Ex1. Calculate the volume of the following rectangular prism:

Volume = l x w x h = 8 cm x 5 cm x 3 cm

= **120 cm3**

* To calculate the volume of other prisms, we use the formula :

Volume = *Ah* where A = base area h = height

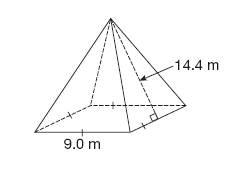
Ex.2 Calculate the volume of the following triangular prism :

\**note : The base shape is a triangle, the height is the part that connects the two base shapes.*

Volume = *Ah =*  ( ½ x 6 x 8) (5) = **120 m3**

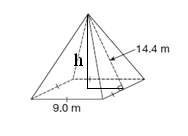
The volume of a **right pyramid** can be calculated using the following formula :

 Where  *A =* area of the base h = height



Ex3. What is the volume of the following right square pyramid to the nearest cubic meter?

\*First, we need to calculate the HEIGHT of this pyramid.

Use Pythagoras theorem!

4.52 + h2 = 14.42

h =  = 

Now, determine the volume:

 = (9.02)( ) = 369.328…. = **369 m3**

* To calculate the volume of a cylinder we use the formula:

 where r = radius and h = height

* To calculate the volume of a cone we use the formula:

 where r = radius and h = height

Ex4. The volume of a cylinder is 450 mm3.  If the radius is 5 mm, what is the height to the nearest tenth of a mm?

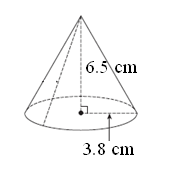


450 = (52)h

Divide both sides by (52)

 = h

h = **5.7 mm**

Ex5. What is the volume to the nearest cubic cm of the following cone?

 = (3.82)(6.5) = 98.289…. = **98 cm3**

**SA and Volume of a Sphere**

Ex.1Given the following sphere, calculate the surface area to the nearest tenth square mm:



SA = 4(3.52) = 153.93…. = **153.9mm2**

* The formula for the volume of a sphere is explained in your text book on page 48.
* 

Ex2. Calculate the volume of the sphere from example 1 to the nearest tenth of a cubic mm.

V = (3.53) = 179.59… = **179.6 mm3**

Ex3. The volume of a sphere is 524 cm3. What is **diameter** of the sphere?



524 = 

Multiply both sides by 3

3(524) = 4

1572 = 4

Divide both sides by 4



125.09….. = r3

Find the cube root of both sides to solve for radius

 = r

r = 5.0012… = 5.0 cm

Diameter =2r = 2(5) = **10 cm**

Ex4. A hemisphere is a sphere cut in half. What is the surface area of a hemisphere with a radius of 3.0 m to the nearest tenth of a square meter?

First, calculate the SA of the sphere:

 = 4(32) = 113.097….m2

Divide by two to get the hemisphere

113.097…./2 = 56.548…. m2

\*This is NOT the area though! Consider, there is a CIRCLE on the top of the hemisphere. The area of this circle needs to be calculated as well!

Acircle = *r 2  =(32) = 28.27….m2*

Add together to get the total SA of the hemisphere!

56.548…. + 28.27…. = 84.823… = **84.8m2**

* To calculate the volume of a hemisphere, we use the formula:



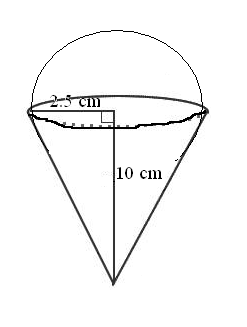
* 

**Composite Solids**

* A composite solid is a 3-D object made up of two or more 3-D objects.
* To calculate the volume of a composite solid, determine the volume of each shape individually and then add together.
* Calculating the surface area of composite solids can be a little more difficult. You have to consider that some of the surfaces may be “hidden” within the object.

Ex1. An ice cream cone with a height of 10 cm and a radius of 2.5 cm is filled with ice cream. A hemisphere of ice cream (with the same radius) sits on top.

1. What is the volume of ice cream to the nearest tenth of a cubic centimetre?



**Sketch it!**

**Volume of a hemisphere =** 

Volume of a cone = 

Total Volume = +  = (2.53) + (2.52)(10)

= 98.174…. = **98.2 cm3**

1. What is the surface area to the nearest square cm of this composite solid?

\*We need to consider the exposed parts only. We have ½ a sphere, and a cone with out the top circle exposed.

SAsphere = 4r2 SAcone = *r2 + r s*

* We know that *r2* from the cone formula represents that circle part. We also know that we have to divide SA of sphere by 2 to get a hemisphere:

SA= 2r2 + *r s*

*Explain to students which each part represents.*

* We also need to calculate the slant height to complete this equation.

a2 + b2 = c2

c2= (2.52) + (102)

c =  = 

* Now we can calculate the SA of the composite solid.

SA = 2(2.52) + (2.5)( ) = 120.2268…. = **120.2 cm2**

Ex2. A stock watering tank is in the shape of a rectangular prism, with a cylinder heater installed as shown in the diagram. Calculate the volume of the tank to the nearest cubic meter when filled with water

1.4 m

3.5 m

cylindrical

heater diameter

= 44 cm

0.8 m

Diameter = 44 cm

First, we need to convert all of our units to the same unit. Convert 44 cm to m.

44 cm x  = 0.44m = diameter

Radius = 0.44/2 = 0.22 m

Now, let’s consider the shapes we have in this composite solid:

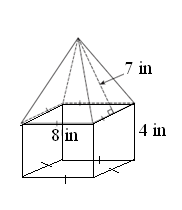
A rectangular prism, and a cylinder.

*Vprism = lwh* 

We need to **subtract** the volume of the cylinder from the volume of the prism.

Vsolid = lwh -  = [(3.5)(1.4)(0.8)] – [(0.222)(1.4)] = 3.70… = **3.7 m3**

*Explain to students how we knew the height of the cylinder was the same as the width of the rectangular prism.*

Ex3. Calculate the surface are of the following composite solid to the nearest square inch.

* Consider the sides that are “hidden”. How many exposed faces are there? (9)

*SApyramid= ½ s (perimeter of base) + (base area) SAprism =2(lw)+2(lh)+2(wh)*

SAobject = ½ s (perimeter of base) + 1(lw)+4(lh)

= ½ (7)(8+8+8+8) + 1(8x8)+4(8 x 4) = **304 in2**

*Explain to students how we developed the formula. Note how the four sides of the prism are of equal length and height.*

Ex4. A cylindrical water tank that is 5.2 m high with a radius of 3.1 m is filled with water. The water drips from the tank into a rectangular prism trough. If the volume of water is completely transferred from the cylinder to the trough, what is the height of the water (to the nearest m) if the length of the trough is 5 m and the width is 2 m?

\*First we need to determine the volume of the cylindrical water tank\*



V = (3.12)(5.2) = 156.991….m2

If this volume is completely transferred to the trough, we will assume the volumes will be the same.

Vtrough= lwh

156.991… = (5)(2)(h)

156.991…. = (10)(h)

Divide both sides by 10

 = h

h=15.699… = **16 m**

**Day 4 - What is Measurement? Introduction to Metric (SI) and Imperial Units Section 1.1 & 1.2**

**LESSON**

*Discuss with students different systems of measurement. What are measurements useful for? What jobs need measurement skills? What are some different units of measurement? What system of measurement do we typically use in Canada? What system of measurement do they use in America?*

**SI measuring system**

**–** “syteme international d’unites” or **metric** – for example meters, centimeters etc.

**-**The base unit for measuring **length** is **the meter (m)**, the base unit for measuring **mass** is the **gram (g)** and the base unit for measuring **volume** is the **litre (L).**

**-**This system is a **DECIMAL** system because it is based on **FACTORS of 10**. Any measurement stated in one SI unit can be converted to another SI unit by multiplying or dividing by a multiple of 10.

**Imperial system**

**–** inches, feet etc. and is most commonly used in the United States and in many trades in Canada.

**-**This system is NOT a decimal system. This system is based on **referents.**

**-** A **referent** is used to estimate a measure; for example, the length from the tip of your thumb to the knuckle is ~ 1 inch.

*\*This measurement system started in Ancient Roman Times. The current Emperor’s foot length would be the standard unit for measuring length (distances). The length of a foot was later standardized to equal 12 inches.\**

-A fraction of an imperial measure of length is usually written in fraction form, not decimal form.

Consider the following:



This ruler has 16 divisions between inches, so the smallest indicated unit is  of an inch.

-The following is an example where measurements with fractions from the imperial system can become quite tricky.

*Show the following youtube video: American Choppers vs The Metric System*

[*http://www.youtube.com/watch?v=Omh8Ito-05M*](http://www.youtube.com/watch?v=Omh8Ito-05M)

*or search for “American Chopper Metric”*

*You may want to discuss with the students the issue; the bearded man was having difficulty properly subtracting his mixed number fractions*

*.*

**Converting Between Metric Units**

We can easily convert between Metric units by multiplying or dividing by 10 each place holder that we move. A helpful way to remember the order of the SI units….

**K H D m d c m**

**K**ing **H**enry **D**anced **m**errily **d**own **c**ountry **m**eadows

**Kilo Hecto Deca \*m, g or L\* deci centi mili**

**BASE UNIT**

The base unit tells us what we are measuring, length (m), mass (g) or volume (L).

Ex. Briana wants to hang curtains in her kitchen. She measures the length of the window to be 250 cm. At the store, the curtain package lists the length in meters. How many meters of curtain will she need?

**250 cm  10 10 = 2.5 m**

Briana will need to buy curtains with a length of 2.5 m.

Ex 2. Margaret is cleaning out her attic and has discovered a box of gold jewelry. She decides to sell it. She determines that she has 0.45 kg of gold. The current price of gold is $8.94 /g. How much money will she make?

0.45 x 10 x 10 x 10 = 450 g

**OR 0.45 x 1000 = 450 g**

4) Now we can multiply by the price of gold to determine how much money she will make: 450 g x $8.94 = $4 023 🡪 wow! ☺

*Discuss some common referents for metric units with students. For example, 1 cm = ~ the length of a fingernail, 1 m = ~ length of on arm from fingers to shoulder*

**Converting Between Imperial Units**

* When measuring length with the imperial system, the following units are used:

**Imperial Unit Relationship Between Units Referent**

Inch (in. or “) Thumb length to knuckle

Foot (ft or ‘) **12 inches = 1 foot** Foot length

Yard (yd) **3 feet = 1 yard** Arm span

Mile (mi) **1760 yards = 1 mile**

Ex. Russell wants to determine how far away the cafeteria is from Mrs. Lin’s classroom in miles. He counts 1584 feet from the classroom to the cafeteria doors. How far is it in miles?

We HAVE **feet** we WANT **miles**

1 mile = 1760 yards , 3 feet = 1 yard. Use **unit analysis** to convert between units.

1584 feet  = **0.3 miles**

* Sometimes measurements will include feet and inches. If we want to convert to one unit only (either feet or inches) we must change one of the measurements to the desired unit.

Ex. Tristan is 5’11” tall. How tall is Tristan in inches?

We know that 5’ = 5 feet and 11” = 11 inches. We WANT **inches** so we must convert 5 feet into inches first.

5 feet x 12  = **60 inches**

We can now add the 11 inches to determine Tristan’s height in inches.

60 inches + 11 inches = **71 inches total**

**Fractions in imperial measurements.**

Ex. LeBron James (a basketball player for the Miami Heat) is 80 inches tall. Convert his height to **Feet and inches**

80 in x = . Write this improper fraction as a mixed number.

* Review with students how to change improper fractions to mixed numbers:
  + How many times will 12 go into 80? (**6)**
  + What is the remainder? (**8)**
  + **Mixed number = 6** 

= 6  ft = 6 ft. 8 in.

\*note: Show students that  ft x 12 in/ft = 8 in.\*

**Map Scales**

Ex. A map of Alberta has a scale of 1: 4 500 000. The distance on the map between Peace River and High Prairie is  in. What is the distance to the nearest mile?

* Review with students how to change a mixed number to an improper fraction:

= 16 x 4 + 11 = 



* The map scale is 1 in represents 4 500 000 in.

() (4 500 000 in) = 21 093 750 in 🡪 now change this to miles.

21 093 750 in x  **= 332.9190341**…. mi

* The distance between Peace River and High Prairie is approximately 333 miles.
* Sometimes we may need to convert from SI units to Imperial, or from Imperial to SI units. To do this, we will require a chart that describes the basic conversion factors:

|  |  |  |  |
| --- | --- | --- | --- |
| **Conversion Factors** | | | |
| **Length** | | **Capacity/ Volume** | |
| *Imperial 🡪 Metric* |  | *Imperial 🡪 Metric* |  |
| 1 in 2.5 cm |  | 1 gal 4.5 L |  |
| 1 ft 0.30 m  1 ft 30 cm |  | **Mass/Weight** | |
| 1 yd  0.9 m  1 yd  90 cm |  | *Imperial 🡪 Metric* |  |
| 1 mi 1.6 km |  | 1 oz 28.35 g |  |
|  | | 1 lb 0.454 kg |  |
| 1 lb 454 g |  |

*\*note: These conversions are approximate. oz = ounce, lb = pound, gal = gallon*

To convert between units, we will use proportions:

1) Set up a fraction :  Place an “x” and the units for what you WANT.

2) Find the appropriate conversion on the chart.

3) Set up a second fraction, with the units of what you WANT in the numerator and what you HAVE in the denominator

4) Perform opposite operations to solve for your unknown.

Ex1. Carlos measures 14 cm but he needs to know this length in inches. How many inches is 14 mm?

14mm = \_\_\_\_\_\_\_in ?

Ex2. You are on vacation in Hawaii and are going on a mountain hike. You read in the guide book that the hike is 3.5 miles. How many km is this?

* WANT **km** HAVE **mi**
* 
* we are converting *Imperial 🡪 metric* so we will use: 1 mi = 1.6 km
* ****

**Multiply both sides by 3.5 mi** **5.6 km**

* We may need to complete more than one conversion if what we want is not on the conversion factor chart.
* For example, you may need to change from Imperial 🡪SI first , then convert the SI units to the desired unit.

Ex3. At least once a year, a truck will get stuck on the High Level Bridge in Edmonton. The bridge has a low clearance of 10’6”. A truck driver knows that her semitrailer is 3.3 m high. Will her vehicle fit under the bridge? Or will she be stopping traffic?

*Show the video “Gimli Glider” on youtube.com – a story about an Air Canada jet that had to crash land in Gimli, Manitoba because they ran out of fuel.*

*The video can be found here:* <http://www.youtube.com/watch?v=hl8foT-v6Vg>

*Or you can search for “Gimli Classroom Version” by SalMathGuy. There are two parts, totaling ~20 minutes.*

*After the video, discuss with the class the implications of properly converting Imperial and Metric units. The fuel technicians did not do the proper conversion, and as such, they only fueled the jet with half as much fuel as it would need (the conversion that the students did before the video). They actually needed 22 300 kg of fuel. How many pounds of fuel is this? (~****49 171.5 lbs)*** *which is twice as much as the plane was given!*